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Math 131 - Fall 2023 - Common Final Exam, version A

Print name:________________________________

Print instructor's name:________________________________

Directions:

- This exam has 17 questions worth a total of 96 points.
- Fill in your name and instructor's name above.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator which cannot connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.

Good luck!

Question:	1	2	3	4	5	6	7	8	9
Points:	6	6	5	6	6	6	6	6	6
Score:									

Question:	10	11	12	13	14	15	16	17	Total
Points:	6	6	4	5	6	6	5	5	96
Score:									

Do not open this exam until you are told to do so.

1. Consider the function $f(x) = \dfrac{8x^2 - 35}{1 - 10x^2}$.

(a) (3 points) Evaluate $\lim_{x \to \infty} f(x)$.

(a) ____________

(b) (3 points) Evaluate $\lim_{x \to 0} f(x)$.

(b) ____________

2. The following table gives some values for $s(t)$, the position of an object (in km) after t minutes.

t	3	4	5	6	7
$s(t)$	24.8	42.0	43.5	46.2	47.0

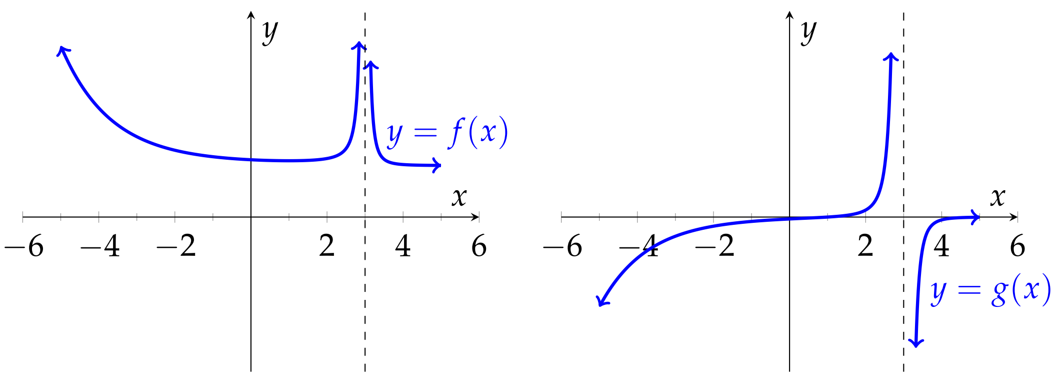
(a) (3 points) Find the average velocity of the object from $t = 4$ to $t = 7$.

The average velocity (with units) is ____________.

(b) (3 points) Estimate the instantaneous velocity at $t = 6$. Show some work that supports your answer.

According to the work above, the instantaneous velocity (with units) is ____________.

3. The graphs below show functions f on the left and g on the right.



(a) (2 points) Choose the option below which best describes the relationship between these graphs.

- ◯ f is the derivative of g.
- ◯ g is the derivative of f.
- ◯ Neither function is the derivative of the other.

(b) (3 points) Support your choice above with a complete sentence which includes at least one fact about slope or concavity at a point.

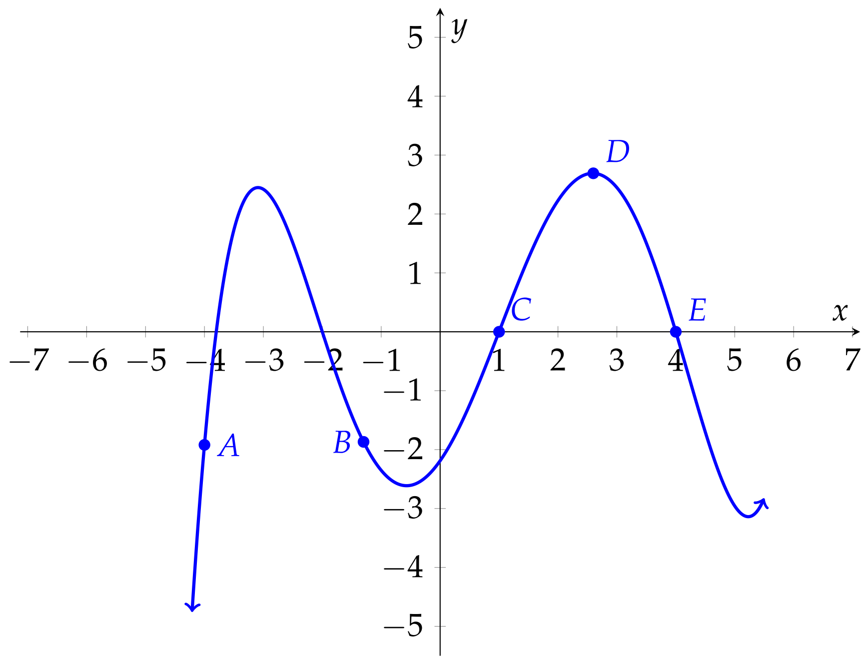
4. Consider the function $f(t) = \cos\big(\ln(t)\big)$.

(a) (3 points) Write the definition of $f'(t)$ as a limit involving h. A formula for the derivative using shortcut rules is not worth credit here. You must use the definition of the derivative. **Do not evaluate the limit.**

(b) (3 points) Estimate $f'(3)$ by evaluating the formula in your limit at an appropriate value of h.

Based on the computations above, $f'(3) \approx$____________

5. Use the graph of $p(x)$ below to answer the following questions. Your answers should list the letters of all labeled points which apply, or "NA" if no labeled points apply.



(a) (2 points) At which point(s) is $p'(x)$ negative?

(a) ____________

(b) (2 points) At which point(s) is $p''(x)$ approximately zero?

(b) ____________

(c) (2 points) At which points is $p''(x)$ positive?

(c) ____________

6. The quantity of a drug in a patient's bloodstream (in mg) t minutes after an injection is $C(t)$.

(a) (3 points) Give the practical meaning of $C(3) = 250$ in a sentence with correct units.

___

(b) (3 points) Give the practical meaning of $C'(3) = -20$ in a sentence with correct units.

___

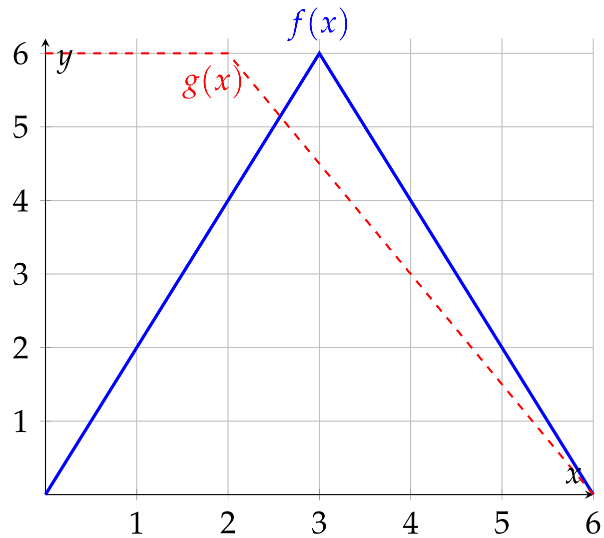
7. (6 points) Find $g''(1)$ if $g(x) = 2\sqrt{x} + x^8 - (5x+1)^2$. Show all your steps.

$g''(1) =$________________________.

8. (6 points) Find the equation of the tangent line to the graph of $w(x) = \dfrac{5x+3}{x-1}$ at $x = 0$. Give your answer in slope-intercept form.

The tangent line is $y =$______________.

9. (6 points) If $r(x) = f(x) \cdot g(x)$, use the graphs of f and g below to evaluate the following derivatives.



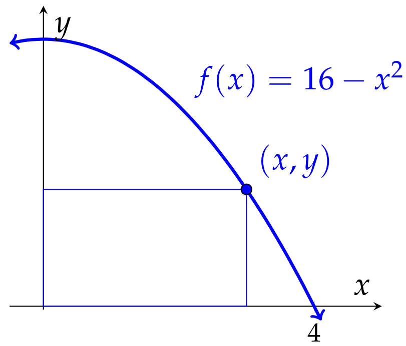
(a) Evaluate $r'(1)$

$r'(1) =$ ____________

(b) Evaluate $r'(4)$.

$r'(4) =$ ____________

10. (6 points) Below is a rectangle inscribed under the graph of f.



Find the value of x on the interval $[0, 4]$ which gives the largest area of this rectangle. Your answer must make clear

- The function of x which you are maximizing,
- the domain of this function, and
- show that your answer gives a maximum value for the area.

According to the work above, the maximum area occurs at $x =$____________.

11. Christopher runs a custom computer design business. He calculates his monthly revenue using the function

$$R(q) = 1800 \ln(20q + 40)$$

and his monthly costs using the function

$$C(q) = 4000 + 40q.$$

(a) (4 points) Find the quantity of computers q he needs to sell to maximize his profit.

He will maximize his profit by making $q =$ ____________ computers.

(b) (2 points) What is the profit he expects to make by selling the quantity of computers determined in part (a). Round your answer to the nearest dollar.

The maximum profit is ____________.

12. (4 points) Let $F(x)$ be an antiderivative of $f(x)$. If $\int_{-3}^{1} f(x)\, dx = 4$ and $F(1) = 5$, find $F(-3)$.

$F(-3) =$ ____________.

13. (5 points) Find the indefinite integral

$$\int \left(4x^4 - \frac{4}{x} + \frac{9}{x^6}\right) dx.$$

Use C for the constant of integration. Show all steps.

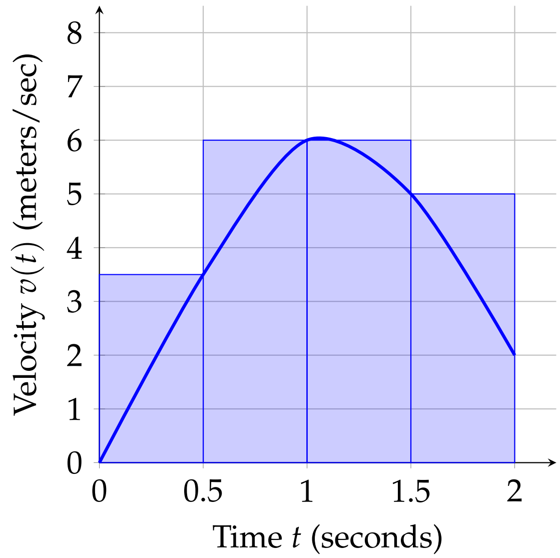
The indefinite integral is ________________________.

14. (6 points) Find the antiderivative $G(x)$ of $g(x)$ with $G(0) = 2$, where

$$g(x) = 2e^x - \sin(x) + 6x + 4.$$

$G(x) =$____________.

15. The figure shows the graph of the velocity (in meters per second) of a particle for $0 \leq t \leq 2$ and the rectangles used to estimate the distance traveled by a Riemann sum.



(a) (1 point) The rectangles represent

- ◯ a left Riemann sum.
- ◯ a right Riemann sum.
- ◯ some other kind of Riemann sum.

(b) (1 point) The estimate for distance traveled based on the indicated Riemann sum is

- ◯ an overestimate.
- ◯ an underestimate.
- ◯ either an underestimate or an overestimate; it is not possible to tell.

(c) (1 point) What is the value of n for the Riemann sum?

(c) ____________

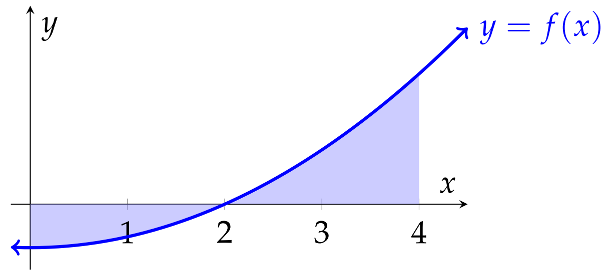
(d) (1 point) What is the value of Δt for the Riemann sum?

(d) ____________

(e) (2 points) Use the rectangles to estimate the total distance traveled by the particle for $0 \leq t \leq 2$.

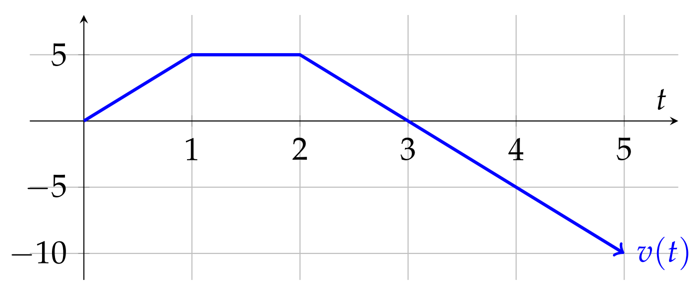
The distance traveled (with units) is approximately ________________________.

16. (5 points) Below is the graph of $y = f(x)$.



Write an expression with an integral or integrals which gives the area of the shaded region. Do not evaluate your expression.

17. (5 points) The graph below gives the velocity of an object t seconds after it begins moving along a line. Use the graph to answer the following questions.



(a) Is the object farther from its starting point at $t = 1$ or $t = 4$? Choose the best answer below.

- ◯ The object is farther from its starting point at $t = 1$.
- ◯ The object is farther from its starting point at $t = 4$.
- ◯ The object is the same distance from its starting point at $t = 1$ and at $t = 4$.

(b) Use some information from the graph to explain in a sentence why your choice is correct.

Elementary Tools from Algebra and Geometry

Quadratic Formula: $ax^2 + bx + c = 0 \implies x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Triangle Area = $\dfrac{1}{2}$ base $\times$ height.

Rectangle Area = base $\times$ height

Five derivative rules for operations on functions.

Constant Multiples: $\dfrac{d}{dx}\Big(cf(x)\Big) = cf'(x)$

Sums & Differences: $\dfrac{d}{dx}\Big(f(x) \pm g(x)\Big) = f'(x) \pm g'(x)$

Product Rule: $\dfrac{d}{dx}\Big(f(x) \cdot g(x)\Big) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\dfrac{d}{dx}\Big(\dfrac{f(x)}{g(x)}\Big) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Chain Rule: $\dfrac{d}{dx}\Big(f(g(x))\Big) = f'(g(x)) \cdot g'(x)$

Ten derivative rules for functions

Derivative of a Constant: $\dfrac{d}{dx}(c) = 0$

The Power Rule: $\dfrac{d}{dx}\Big(x^n\Big) = nx^{n-1}$

Exponential Functions: General Case: $\dfrac{d}{dx}\Big(a^x\Big) = a^x \cdot \ln(a)$ Special Case: $\dfrac{d}{dx}\Big(e^x\Big) = e^x$

Three Trigonometric Rules:

$\dfrac{d}{dx}\Big(\sin(x)\Big) = \cos(x)$

$\dfrac{d}{dx}\Big(\cos(x)\Big) = -\sin(x)$

$\dfrac{d}{dx}\Big(\tan(x)\Big) = \sec^2(x) = \dfrac{1}{\cos^2(x)}$

Three Inverse Function Rules:

$\dfrac{d}{dx}\Big(\ln(x)\Big) = \dfrac{1}{x}$

$\dfrac{d}{dx}\Big(\arctan(x)\Big) = \dfrac{1}{1+x^2}$

$\dfrac{d}{dx}\Big(\arcsin(x)\Big) = \dfrac{1}{\sqrt{1-x^2}}$

General Antiderivative Rules

If k is a constant $\int k\,dx = kx + C$

$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$, when $n \neq -1$

$\int a^x\,dx = \dfrac{a^x}{\ln(a)} + C$

$\int e^x\,dx = e^x + C$

$\int \cos(x)\,dx = \sin(x) + C$

$\int \sin(x)\,dx = -\cos(x) + C$

$\int \sec^2(x)\,dx = \tan(x) + C$

$\int \dfrac{1}{x}\,dx = \ln(|x|) + C$

$\int \dfrac{1}{1+x^2}\,dx = \arctan(x) + C$

$\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$